BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2022

Senior Preliminary

April 2022

1. The number of perfect cubes from 1 through 1,000,000 that are multiples of 7 is:

   (A) 12 (B) 13 (C) 14 (D) 15 (E) 16

2. There are three possible Math electives to take. Alice, Bob and Carol each randomly select an elective. The probability that they all choose different electives is:

   (A) $\frac{1}{9}$ (B) $\frac{2}{9}$ (C) $\frac{3}{9}$ (D) $\frac{4}{9}$ (E) $\frac{6}{9}$

3. In a certain city, a taxi charges 0.20\$ per 1/5 km traveled when moving faster than x km/h. It charges 0.15\$ per minute when moving slower than x km/h. At x km/h, both methods of charging produce the same cost to the rider. The value of x is:

   (A) 9 (B) 10 (C) 12 (D) 15 (E) 18

4. Given that $y^2 + 6x^2 = 5xy$, the largest value of $\dfrac{y+5x}{6x+y}$ is:

   (A) 0 (B) $\frac{5}{6}$ (C) $\frac{7}{8}$ (D) $\frac{8}{9}$ (E) $\frac{6}{5}$

5. Consider the sequence $1, 2, 2, 3, 3, 3, 4, 4, 4, 4, \ldots$, where each integer n appears n times. The 2020th term in this sequence is:

   (A) 60 (B) 61 (C) 62 (D) 63 (E) 64

6. If the following equations are true

   $$\begin{aligned} A + B &= 1 \\ B + C &= 2 \\ C + D &= 3 \\ D + E &= 4 \\ E + F &= 5 \\ &\vdots \\ Y + Z &= 25 \end{aligned}$$

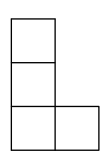
   then $A + Z$ equals:

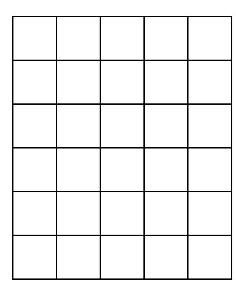
   (A) 14 (B) 13 (C) 12 (D) 11 (E) 10

7. The cutout shown is used to cover exactly four of the squares on the 5×6 checkerboard shown on the right. If rotations of the cutout are allowed, but not reflections, then the number of different choices for the four squares covered is:

   (A) 56 (B) 58 (C) 60

   (D) 62 (E) 64





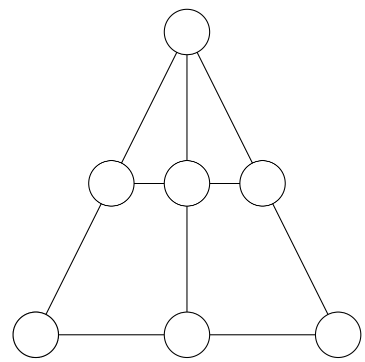
8. A circular robotic vacuum cleaner with diameter 0.5 m moves around a room in the shape of a square 6 m on a side. The fraction of the floor area of the room that it can clean is:

   (A) $\frac{576-\pi}{576}$ (B) $\frac{572+\pi}{576}$ (C) $\frac{140+\pi}{144}$ (D) $\frac{144-\pi}{144}$ (E) $\frac{32+\pi}{36}$

9. An number has the “monotonic digits" property if its digits are distinct and either increase from left to right or decrease from left to right. The number of integers between $100,000$ and one million with the “monotonic digits" property is:

   (A) 84 (B) 168 (C) 192 (D) 294 (E) 306

10. The accompanying diagram contains several sets of circles that ”line up” (3 circles to a line). There are 5 such ”lines”. The integers from 1 through 7 are to be inserted, one number to a circle, so that the sum of the three numbers in each line is the same (this can be done in many ways). The number that can **not** be placed in the lower left circle is:



    (A) 1 (B) 2 (C) 3

    (D) 4 (E) 5

11. Three faces of a rectangular box meet at one corner of the box. The centres of those faces are the vertices of a triangle having sides of lengths 4, 5 and 6 cm. The volume of the box (in cm^3) is:

    (A) $45\sqrt{3}$ (B) $45\sqrt{6}$ (C) $90\sqrt{6}$ (D) 125 (E) $120\sqrt{2}$

12. A function f is said to be “green" if and only if the following properties hold:

    (i) $f(x)$ is defined if and only if x is an integer.
    (ii) $f(x) + f(y) = f(x+y) - xy$ for all integers x and y;
    (iii) $f(1) = c$ where c is some positive integer;
    (iv) $f(n) = 2020$ for some integer $n \geq 2$.

    The number of possible “green" functions is:

    (A) 0 (B) 1 (C) 2 (D) 3 (E) 4