

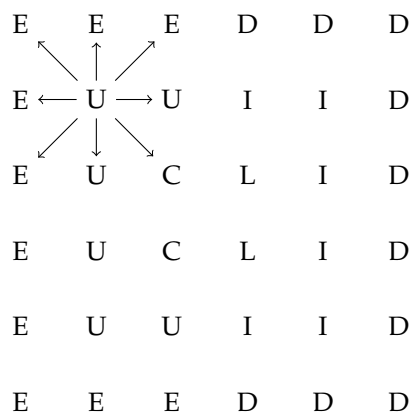
# BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2023

## Senior Final, Part B

May 5, 2023

1. The word EUCLID can be spelled by tracing paths through the given array of letters.

As shown in the diagram, steps to adjacent letters horizontally, vertically, or diagonally are allowed.



Determine the number of different paths which spell the word EUCLID.

2. What number leaves the same non-zero remainder when divided into 1108, 1453, 1844, and 2258?
3. Raul rolls two normal 6-sided dice, each die showing  $\{1, 2, 3, 4, 5, 6\}$ , and adds the numbers showing on them to get a sum of at least 2 and at most 12. He then raises the sum to the power of 4. The result is a number whose units digit is a 6.
  - (a) What is the probability that the original sum was 8?
  - (b) What is the probability that at least one of the dice that he rolled showed a 4?
4. The three points  $(-7, 1)$ ,  $(0, 8)$ , and  $(18, -4)$  in the Cartesian plane lie on the boundary of a circle. Determine the location of the centre of the circle.
5. If for any real number  $x$ , we have that  $xf(x) + f(1 - x) = x^2 + 2$ . Find each of the following.
  - (a)  $f(0)$
  - (b)  $f(5)$
  - (c)  $f(19)$
  - (d) A formula for  $f(x)$
  - (e) Show that  $f$  is unique.