BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2018

Junior Final, Part A Problems & Solutions

1. A group of girls is standing in a circle. They are evenly spaced and numbered in order. The 5th girl is directly opposite the 17th girl. The number of girls in the circle is:

   (A) 22 (B) 24 (C) 25 (D) 26 (E) none of these

   Solution
   Each girl is paired with her “direct opposite". Moving backwards, if the 5th girl is opposite the 17th, then the 4th of opposite the 16th and so on. Thus the first girl is opposite the 13th. This tells us there are 12 pairs, and hence there are 24 girls in total.

   Answer: B

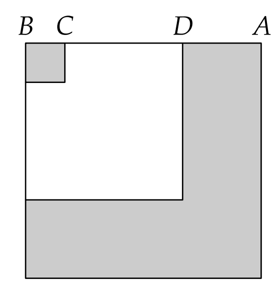
2. There are n coins in a jar. One-half of them are quarters, one-third of them are dimes, and one-sixth of them are nickels. If ten dimes are added to the jar, the number of dimes equals the number of quarters. The value of n is:

   (A) 10 (B) 20 (C) 30 (D) 60 (E) 120

   Solution
   We're given that the number of quarters is $\frac{1}{2}n$, and the number of dimes is $\frac{1}{3}n$, and $\frac{1}{2}n = \frac{1}{3}n + 10$. Thus $\frac{1}{6}n = 10,\ n = 60$. You can also use the “trial and error" approach on the choices, where one would find that if $n = 60$, the number of quarters is 30 and the number of dimes is 20, which is seen to work.

   Answer: D

3. There are three overlapping squares in the diagram. Also, $BA = 2CD = 6BC$. The ratio of the shaded area to the unshaded area is:

   (A) 7 : 5 (B) 6 : 5 (C) 5 : 4

   (D) 5 : 6 (E) 1 : 1

   Solution
   Using the information given, we can break the diagram into small, equally-sized squares. The ratio of total shaded to total unshaded equals $21 : 15 = 7.5$.

   Answer: A

4. Given the arithmetic sequence $16, 23, 30, 37, 44, \ldots$, a new sequence is formed by subtracting 5 from each term. A number that appears in the new sequence is:

   (A) 221 (B) 222 (C) 223 (D) 224 (E) 225

   Solution
   The new sequence $11, 18, 25, 32, 39$ are number with remainder 4 when divided by 7, and 221 will as well.

   Answer: A

5. A hot dog costs half as much as a hamburger. If the price of a hot dog rises 5% and the price of a hamburger rises 10%, then the increase in cost of buying three hot dogs and three hamburgers is:

(A) $7\frac{1}{2}\%$ (B) 8% (C) $8\frac{1}{3}\%$ (D) $8\frac{1}{2}\%$ (E) 9%

Solution

If hot dogs and hamburgers cost the same amount, then if we are buying the same number, then the percentage increase is the average increase $7\frac{1}{2}\%$. However, since hamburgers cost twice as much, the price increase has twice as much impact. So $\frac{.05+.10+.10}{3} = .08\overline{3} = 8\frac{1}{3}\%$.

Answer: C

6. You have five cubes: one red, one yellow, one green, one light blue and one dark blue. The number of ways in which the five cubes can be arranged in a single stack, without the blue cubes touching, is:

(A) 24 (B) 48 (C) 72 (D) 96 (E) 120

Solution

There are $5! = 120$ ways to stack the cubes total. If the blue cubes are view as one cube, there are $4! = 24$ ways to stack the cubes. Given that there are 2 different orders for the blue cubes, there are $2 \times 24 = 48$ different stacks where the blue cubes occur together. Thus the number stacks where the blue cubes do not occur together is $120 - 48 = 72$.

Answer: C

7. An operation $\star$ for numbers is defined as $a \star b = \dfrac{a+2b}{2}$. The value of $4 \star (2 \star 3)$ is:

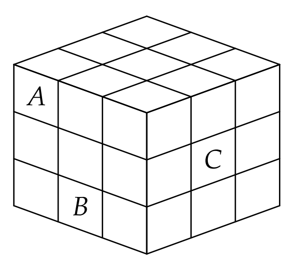
(A) 5 (B) 5.5 (C) 6 (D) 6.5 (E) 7

Solution

$2 \star 3 = \frac{2+2(3)}{2} = \frac{2+6}{2} = 4$. Also, $4 \star 4 = \frac{4+2(4)}{2} = \frac{4+8}{2} = 6$.

Answer: C

8. A large cube is made up of 27 smaller cubes, each of side length one centimetre, as shown in the diagram. If the three cubes marked A, B, and C are removed, then the total surface area (in cm^2) of the object that remains is:



(A) 66 (B) 60 (C) 58

(D) 56 (E) 54

Solution

The surface area of the cube is 9 cm^3 per face; that is a total face area of $9 \times 6 = 54$ cm^2. Removing A won't change the surface area, removing B with all 2 cm^2, and removing C will add 4 cm^2. So $54 + 6 = 60$ cm^2.

Answer: B

9. There are n students in a gym class. Each is wearing a shirt (either red or blue) and shorts (also either red or blue). There are exactly 10 students wearing a red shirt, exactly 12 students wearing red shorts, and exactly 14 students wearing a shirt the same color as their shorts. The smallest possible value of n

is:

(A) 16 (B) 18 (C) 20 (D) 22 (E) 24

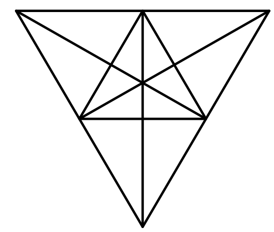
Solution

If we have the largest overlap of red shirts and red shorts, then all 10 students wearing red shirts also have red shorts and red shorts. Then, there are $12 - 10 = 2$ students in red shorts and blue shirts, and $14 - 10 = 4$ students in both blue shirts and blue shorts. The answer is $10 + 2 + 4 = 16$.

Answer: A

10. The total number of triangles appearing in the diagram is:

(A) 35 (B) 38 (C) 41 (D) 44 (E) 47



Solution

There are a number of systematic ways to classify and then count the triangles. It is a good idea to do it in two distinct ways in order to lessen the likelihood of a counting error.

One way to classify the triangles in the figure is by the number of distinct line segments crossing the interior of the triangle. This process obviously includes all of the triangles in the figure and does not include any of them twice. Careful examination of the figure reveals there are 12 triangles with no lines passing through them. Similarly, there are $12, 6, 9, 0, 0, 7$ triangle(s) for each of the cases of $1, \ldots, 6$ lines, and 1 for the final (non-zero) case

Another way to classify the triangles is by the congruency class of the triangle. To this end, for ease of identification, label all of the vertices of the triangles in the figure with the numbers 1, ..., 9 beginning at top left and proceeding by rows from left to right so that 5 is in the middle and 9 is at the bottom. One can then label the triangles using ordered triples and classify them by their respective congruency classes. A complete list of representatives of the nine congruency classes is

$$(1,2,4),(1,2,5),(1,2,7),(1,2,9),(1,3,5)(1,3,10)(1,9,10),(2,5,4),(2,5,7).$$

Careful consideration of the figure reveals there are, respectively, $12, 6, 4, 6, 3, 1, 6, 6$, and 3 triangles in each class for a total of $12 + 6 + 4 + 6 + 3 + 1 + 6 + 6 + 3 = 47$ triangles.

Answer: E