BRITISH COLUMBIA COLLEGES

Senior High School Mathematics Contest, 2001

Preliminary Round

Wednesday March 7, 2001

1. The digit that must be placed in front of the five digit number 56734 to produce a six digit number that is divisible by 11 is:

   (a) 3 (b) 5 (c) 6 (d) 7 (e) 8

2. The value of x for which $16^{2x+\frac{1}{4}} = 8^{3+2x}$ is:

   (a) 1 (b) 4 (c) 8 (d) 9 (e) 16

3. If the parabola $y = ax^2 + bx + c$ passes through the points $(-1, 12)$, $(0, 5)$, and $(2, -3)$, the value of $a + b + c$ is:

   (a) -4 (b) -2 (c) 0 (d) 1 (e) 2

4. Given that $(-5, 12)$ and $(5, 12)$ are endpoints of a diameter of a circle, another point on the same circle is:

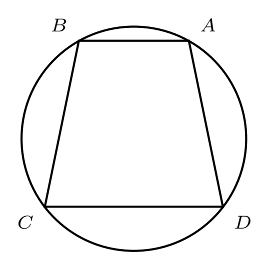
   (a) $(0, 13)$ (b) $(0, 7)$ (c) $(-5, -12)$ (d) $(5, -12)$ (e) $(13, 12)$

5. In a certain algebra class, Chris and Pat count the number of students and compare notes. "Hmm, $\frac{12}{17}$ of my classmates are women," notes Pat. "Funny," replies Chris, "$\frac{5}{7}$ of my classmates are women." They were both right. The number of students in the class is:

   (a) 84 (b) 85 (c) 119 (d) 120 (e) 121

6. An isosceles trapezoid $ABCD$ is inscribed in a circle of radius r, as shown. If $\overline{AB} = 2$, $\overline{CD} = 8$, and $\overline{AD} = 5$, then r equals:

   (a) 4 (b) $4\frac{1}{8}$ (c) $4\frac{1}{2}$ (d) $\sqrt{17}$ (e) $\frac{1}{8}\sqrt{1025}$



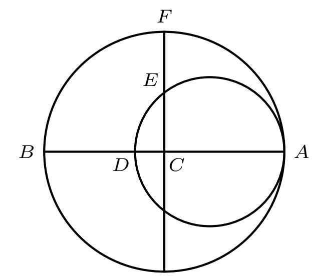
7. Given that $x + y = 1$ and $x^2 + y^2 = 4$, the value of $x^3 + y^3$ is:

   (a) $\frac{5}{2}$ (b) 4 (c) $\frac{11}{2}$ (d) 8 (e) 16

8. Two perpendicular lines intersect at the point $(9, 2)$. If the x-intercept of one line is twice the x-intercept of the other, then a possible sum of the x-intercepts is:

   (a) $\frac{17}{2}$ (b) 10 (c) $\frac{51}{2}$ (d) $\frac{45}{2}$ (e) 5

9. Two circles are tangent to each other at A and the centre of the larger circle is at C. The lines AB and FC are perpendicular diameters of the larger circle. If $\overline{BD} = 9\,\text{cm}$ and $\overline{FE} = 5\,\text{cm}$, then the radius of the smaller circle, in centimetres, is:



(a) 14 (b) 18 (c) $19\frac{1}{2}$
(d) $20\frac{1}{2}$ (e) 21

10. A woman has three daughters, each of whom has three daughters. If they all get together in one room, there are several pairs of cousins. (Two daughters are cousins if their mothers are sisters.) The number of distinct pairs of cousins in the room is:

(a) 9 (b) 18 (c) 27 (d) 45 (e) 54

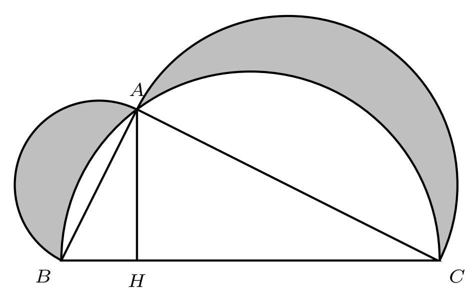
11. The vertices of the quadrilateral $ABCD$ in counterclockwise order are $A(0,0)$, $B(k,0)$, $C(k+m,n)$, and $D(m,n)$, where $k>0, m>0, n>0$. The area of the quadrilateral $ABCD$ is:

(a) mn (b) $n(k+m)$ (c) km (d) kn (e) $k\sqrt{m^2+n^2}$

12. When the bottom of a ladder is 9 metres from the base of a high brick wall with the ladder leaning against the wall, 8 metres of the ladder's length extend beyond the top of the wall. When the bottom of the ladder is 5 metres from the base of the wall with the ladder leaning against the wall, 10 metres of the ladder's length extend beyond the top of the wall. The height of the wall, in metres, is:

(a) 9 (b) 12 (c) 16 (d) 23 (e) 24

13. A semicircle BAC is mounted on the side BC of the triangle ABC. Semicircles are also mounted outwardly on the sides BA and AC, as shown in the diagram. The shaded crescents represent the area inside the smaller semicircles and outside the semicircle BAC. If the diameter BC of the semicircle BAC is 10 cm and the altitude AH is 4 cm, then the total shaded area, in square centimetres, is:



(a) 5π (b) 6π (c) 18
(d) 19 (e) 20

14. Let $f_0(x) = \dfrac{1}{1-x}$ and $f_{n+1}(x) = f_0\left(f_n(x)\right)$. Then the value of $f_{100}(3)$ is:

(a) $-\dfrac{1}{2}$ (b) $\dfrac{2}{3}$ (c) 3 (d) $\dfrac{3}{2}$ (e) -2

15. Let $a \circledL b$ represent the operation on two numbers a and b, which selects the larger of the two numbers, with $a \circledL a = a$. Let $a \circledS b$ represent the operation which selects the smaller of the two numbers with $a \circledS a = a$. Of the three rules:

1. $a \circledL b = b \circledL a$
2. $a \circledL (b \circledL c) = (a \circledL b) \circledL c$
3. $a \circledS (b \circledL c) = (a \circledS b) \circledL (a \circledS c)$

the correct rule(s) is (are):

(a) none (b) 1 only (c) 1 and 2 only (d) 1 and 3 only (e) all three