

BRITISH COLUMBIA COLLEGES

Senior High School Mathematics Contest, 2002

Final Round, Part A

May 3, 2002

1. A certain two-digit number is divided by the sum of its digits and the quotient is 7 with no remainder. The number of two-digit numbers with this property is:

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

2. A rectangular sheet of paper $ABCD$ is 12 centimetres wide and 16 centimetres long. The paper is folded so that two diagonally opposite corners, A and C , coincide. The length of the fold, in centimetres, is:

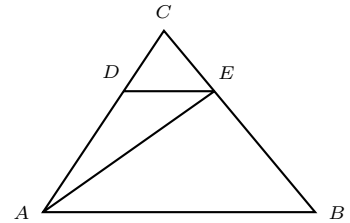
(a) $\frac{25}{2}$ (b) $\frac{76}{5}$ (c) $10\sqrt{2}$ (d) 15 (e) none of these

3. The expression $\frac{x^2 - 9}{x - 4}$ is positive for:

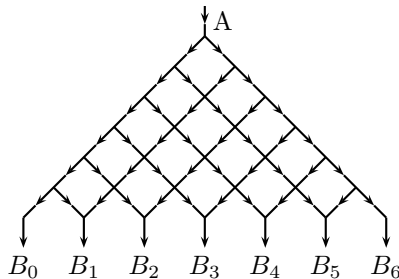
(a) all $x < 3$ (b) all $x > 3$ (c) all $x > -3$
(d) all $x > 4$ or strictly between -3 and 4 (e) all $x < -3$ or strictly between -3 and 4

4. In the diagram sides AB and DE are parallel and $DE : AB = 1 : 3$. If the area of triangle CDE is 20, then the area of the triangle DEA is:

(a) 20 (b) 40 (c) 80
(d) 100 (e) 120



5. Consider the device below which is sometimes used to illustrate a particular probability distribution.



The device consists of a system of paths as shown. Steel balls are fed through the entrance at A and then fall downwards. At every “fork”, approximately half of the balls go right while the other half go left. As a result, a ball can exit through any one of the seven exits, $B_0, B_1, B_2, B_3, B_4, B_5$, or B_6 . If 64 balls enter through A , then the number of balls that exit through B_3 is approximately:

(a) 8 (b) 9 (c) 16 (d) 20 (e) 32

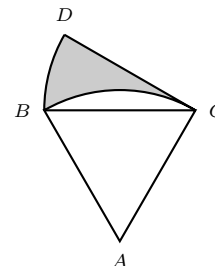
6. A circle of radius 8 is inscribed in an isosceles trapezoid. The length of the larger base of the trapezoid is 20. The area of the trapezoid is:

(a) $\frac{1312}{5}$ (b) 360 (c) 240 (d) 288 (e) $180\sqrt{2}$

7. A polyhedron is a solid bounded by plane (flat) surfaces only. These are the faces of the polyhedron. For example, a pyramid with a square base is a polyhedron with five faces and a cube is a polyhedron with six faces. A certain polyhedron has faces that are either equilateral triangles or squares. The squares and the triangles all have sides of the same length. The number of different such polyhedra that can be constructed having exactly five faces is:

(a) 0 (b) 1 (c) 2 (d) 3 (e) more than 3

8. In the diagram ABC is an equilateral triangle with side length 2 units, arc BC is centered at point A , and the line segment CD is tangent to the arc BC at C and is the radius of arc BD . The shaded area, in square units, is:



(a) $\sqrt{3} - \frac{\pi}{3}$ (b) $\frac{2\pi}{3} - \sqrt{3}$ (c) $\frac{\pi}{3}$
 (d) $\frac{2\pi}{3}$ (e) $\sqrt{3}$

9. One of the ski lifts at Big White climbs the mountain in a line that parallels one of the ski runs. Both the run and the lift are three kilometres long, and a chair passes the starting point of the lift every 10 seconds. A skier starts her run just as a chair arrives at the top of the lift and another chair starts back down. She arrives at the bottom just as a chair is starting up the mountain and another chair is completing its descent. Counting these two chairs and the two at the start of her run, she sees 51 chairs on their way up the mountain and 21 chairs on their way down. If the chair that started down the slope at the same time the skier did is still on its way down when she reaches the bottom of the run, then the skier's average speed, in kilometres per hour, is

(a) 20 (b) 36 (c) 40 (d) 72 (e) 84

10. Antonino owns three dogs, Winnie, Ariel and Murphy. On any given day two of the dogs are completely honest and will answer any question truthfully. The other dog is an alternating liar. That is, if it answers one question honestly it will lie on the next question. Which dog is the alternating liar changes from day to day, so that on any day Antonino does not know which of the dogs are always telling the truth and which is the alternating liar. One day Antonino realizes that his homework is missing, and he decides that one of the dogs must have eaten it. (Isn't that always the way?). He decides to ask the dogs a series of questions to determine the culprit. The dogs, of course, cannot talk. They can only answer yes or no, where "Arf!" means yes and "Arf! Arf!" means no. The first two questions and the answers are:

Antonino: Ariel, did you eat my homework?
 Ariel: Arf!
 Antonino: Ariel, did you eat my homework?
 Ariel: Arf! Arf!

Of the following five statements the one that must be true is:

- (a) Ariel ate Antonino's homework.
 (b) If Antonino asks Winnie whether Ariel ate his homework, then, no matter what the answer is, he will know who ate it.
 (c) If Antonino asks Winnie whether Ariel ate his homework and Winnie says "Arf!", then he knows that Ariel ate it.
 (d) If Antonino asks Winnie whether Ariel ate his homework and Winnie says "Arf! Arf!", then he knows that Murphy ate it.
 (e) Either Winnie or Murphy ate Antonino's homework.