BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2010

Senior Final, Part A

Friday, May 7

1. A circular tabletop 2 metres in diameter stands 1 metre above the floor. A square tablecloth laid on the tabletop just touches the floor at its four corners. The length of one side of the tablecloth, measured in metres, is:

   (A) $2\sqrt{2}$ (B) 4 (C) $4\sqrt{2}$ (D) 4π (E) 8

2. The number of different prime factors of the value of $2009^2 + 2 \times 2009 + 1$ is:

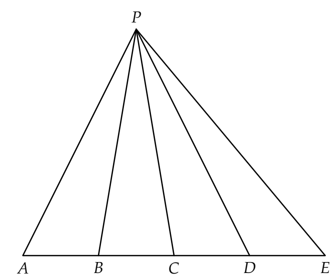
   (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

3. A company wants to construct a rectangular box that will hold exactly 150 cubes each of dimension $1 \times 1 \times 1$ centimetre. The minimum possible surface area of the box, measured in square centimetres, is:

   (A) 120 (B) 160 (C) 170 (D) 190 (E) 230

4. The area of triangle APE shown in the diagram is 12. If $\overline{AB} = \overline{BC} = \overline{CD} = \overline{DE}$, then the sum of the areas of all the triangles that appear in the diagram is:

   (A) 24 (B) 36 (C) 42
   (D) 48 (E) 60



5. The sum of all positive integers less than 1000 that are divisible by 9 or 11, but not both is:

   (A) 90099 (B) 95544 (C) 100989 (D) 106434 (E) 110888

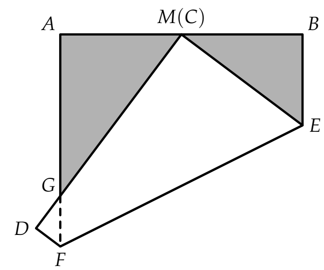
6. Six mattresses are stacked in a warehouse. Each mattress was originally 12 cm thick, but thickness is reduced by one third each time an additional mattress is piled on top. The height of the stack, in cm, is closest to:

   (A) 24 (B) 27 (C) 30 (D) 33 (E) 36

7. In the plane, the angles of a regular polygon with n sides add up to less than n^2 degrees. The smallest possible value of n satisfies:

   (A) $n < 40$ (B) $40 \le n < 80$ (C) $80 \le n < 120$
   (D) $120 \le n < 160$ (E) $n \ge 160$

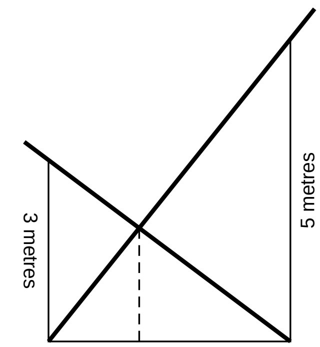
8. A square sheet of paper $ABCD$ is 24 cm on a side. Point M is the midpoint of side AB. Vertex C is folded up to coincide with M, as shown. The length, measured in centimetres, of the segment AG is:



(A) 15 (B) 16 (C) 17

(D) 18 (E) 20

9. Workers are digging a trench with a flat bottom on the side of a hill. The width of the trench is not known, but the trench is 5 metres deep on the uphill side and 3 metres deep on the downhill side. Two ladders are lowered into the trench from opposite sides, as shown in the diagram. Each ladder touches one corner of the trench and the edge of the opposite side on the top. The distance measured in metres from the bottom of the trench to the point of intersection of the ladders is:



(A) $\frac{11}{12}$ (B) $\frac{5}{3}$ (C) 2

(D) $\frac{15}{8}$ (E) $\frac{7}{3}$

10. Charles has a digital watch that he has set to 24 hour mode, but the colon (:) dividing the hours and minutes is broken. When the watch just changes from 2009 to 2010 Charles looks up at the analog clock on the wall, which is synchronized exactly with his watch, and notices that the hour and minute hand almost form a straight line. He estimates that it will be no more than 1 minute before they form a straight line. The number of seconds, to the nearest second, that Charles must wait until the hour and minute hands form a straight line is:

(A) 5 (B) 10 (C) 35 (D) 50 (E) 55

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Senior Final, Part B

Friday, May 7

1. (a) If $r^2 - r - 10 = 0$, determine the value of $(r+1)(r+2)(r-4)$.

   (b) Determine the number of odd positive integers n for which $11 \cdot 14^n + 1$ is a prime number. Justify your answer.

2. Find the five distinct integers for which the sums of each distinct pair of integers are the numbers 0, 1, 2, 4, 7, 8, 9, 10, 11, and 12.

3. Let

$$F(x) = \frac{1}{\sqrt{x + 2\sqrt{x-1}}} + \frac{1}{\sqrt{x - 2\sqrt{x-1}}}$$

   The value of $F\left(\frac{3}{2}\right)$ is an integer. Find the integer.

4. The awards officer at a certain college receives scholarship applications from five students. Each student is awarded a scholarship in a different amount. Letters are sent out to each of the five students announcing his or her award, but the letters are accidentally mixed up so that all but one student receives the letter for a different student. Determine the number of ways in which this can happen.

5. As shown, the large unshaded circle is tangent to the semicircle at the point A and to the diameter of the semicircle at point B, the midpoint of the diameter. The two smaller unshaded circles are tangent to the semicircle, the diameter, and the large unshaded circle. What fraction of the semicircle is shaded.