# BRITISH COLUMBIA SECONDARY SCHOOL MATHEMATICS CONTEST, 2008

## Senior Final, Part A

**Friday, May 2**

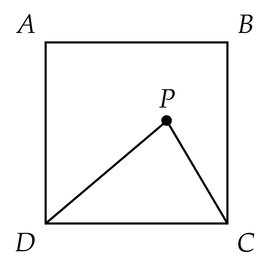
1. Define $(2n+1)! = 1 \times 2 \times 3 \times \cdots \times (2n+1)$, the product of all the positive integers from 1 to $2n+1$. Define $(2n+1)¡ = 1 \times 3 \times 5 \times \cdots \times (2n+1)$, the product of all the odd positive integers from 1 to $2n+1$. Then the expression

$$\frac{(2n+1)!}{(2n+1)¡}$$

is equal to:

(A) $2^n n!$ (B) $2\,(n!)$ (C) $(2n)!$ (D) 1 (E) $2n+1$

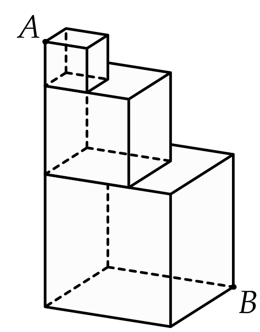
2. $ABCD$ is a square of side 1. A point $P$ is chosen at random inside the square and is connected to points $C$ and $D$ to form the triangle $PDC$ as shown in the diagram. The probability that the measures of all of the angles in triangle $PDC$ are less than $90°$ is:




(A) $\dfrac{\pi}{8}$ (B) $\dfrac{7\pi}{8}$ (C) $\dfrac{\pi}{2}$

(D) $\dfrac{8-\pi}{8}$ (E) $\dfrac{1}{3}$

3. A $1 \times 1 \times 1$ cube, a $2 \times 2 \times 2$ cube, and a $3 \times 3 \times 3$ cube are stacked as shown. The length of the portion of the line segment $AB$ that lies entirely in the $3 \times 3 \times 3$ cube is:




(A) $\sqrt{3}$ (B) $3\sqrt{3}$ (C) $\dfrac{3\sqrt{6}}{2}$

(D) $3\sqrt{6}$ (E) $6\sqrt{3}$

4. The number of pairs of positive integers $(x, y)$ that satisfy the equation

$$x^2 + y^2 = x^3$$

is:

(A) 0 (B) 1 (C) 2 (D) Answer is infinite (E) None of these

5. The number of solutions $(x, y)$ to the inequality

$$|x| + |y| < 50$$

for which both $x$ and $y$ are integers is:

(A) 4851 (B) 4901 (C) 5101 (D) 5050 (E) 9801

6. A square has sides of unit length. A second square is formed whose vertices are the midpoints of the sides of the first square. A third square is formed whose vertices are the midpoints of the sides of the second square, and so on. The sum of the perimeters of all of the squares thus formed is:

   (A) $4\sqrt{2}-4$  (B) $4\sqrt{2}+4$  (C) $4\sqrt{2}+8$  (D) 16  (E) Infinite

7. The expression

$$\left(21^2+22^2+23^2+\cdots+40^2\right)-\left(20^2+19^2+18^2+\cdots+1^2\right)$$

   has a value of:

   (A) 16359  (B) 16400  (C) 16441  (D) 19270  (E) 20000

8. The constant term in the expansion of $\left(3x+\dfrac{2}{x^2}\right)^6$ is:

   (A) 15  (B) 324  (C) 4320  (D) 4860  (E) No constant term

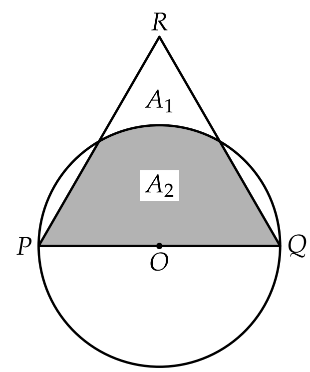
9. Recall that for integers $a$ and $b$, the remainder when $b$ is divided by $a$ is written $a \pmod{b}$. The value of

$$\left[2^0+2^1+2^2+2^3+\cdots+2^{2008}\right] \pmod{15}$$

   is:

   (A) 0  (B) 1  (C) 8  (D) 9  (E) 11

10. In the diagram $PQR$ is an equilateral triangle and $PQ$ is the diameter of the circle centred at $O$. Area $A_1$ is the area inside the triangle that is outside the circle and the shaded area $A_2$ is the area inside the triangle that is also inside the circle. The ratio of area $A_1$ to area $A_2$ is:




   (A) $\dfrac{2\sqrt{3}}{\pi}$  (B) $\dfrac{\pi-\sqrt{3}}{\pi+\sqrt{3}}$  (C) $\dfrac{1}{2}$

   (D) $\dfrac{2}{3}$  (E) $\dfrac{3\sqrt{3}-\pi}{3\sqrt{3}+\pi}$